Cascading Failures and Energy Landscapes for Power Systems

LANS Seminar

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Argonne
Math • Environment • Power Grid • Rare Events
MACSER

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Contents

Power Grid Problems

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Cascad(a)-ing Failure

"Every time we touch, I feel the static" – Cascada[1], and also, squirrels

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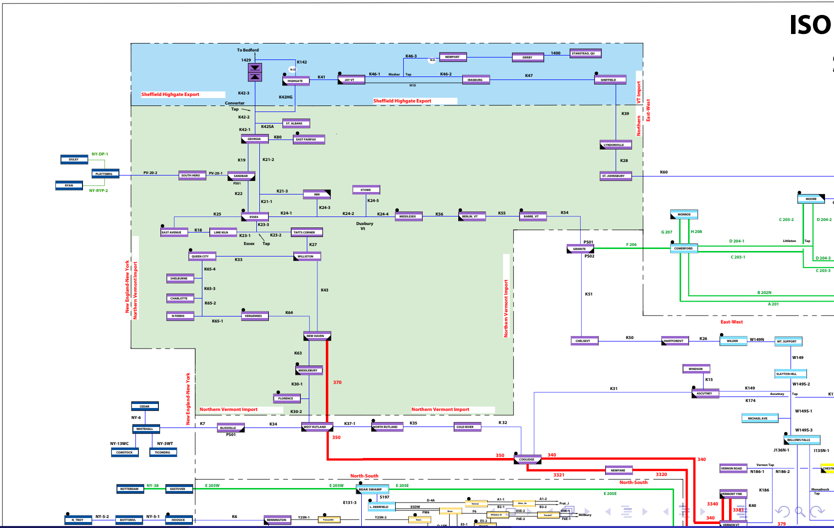
[1] *Every time we touch*, Maggie Reilly, 1992

Foundational Power Grid Problems



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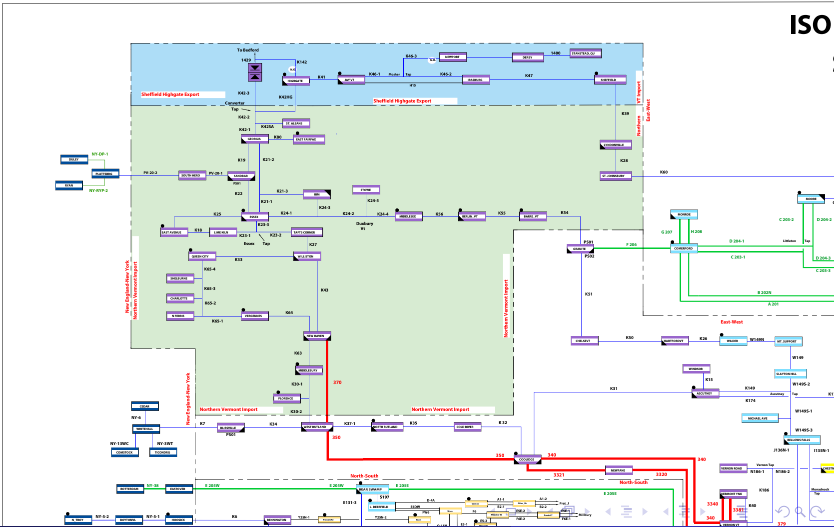

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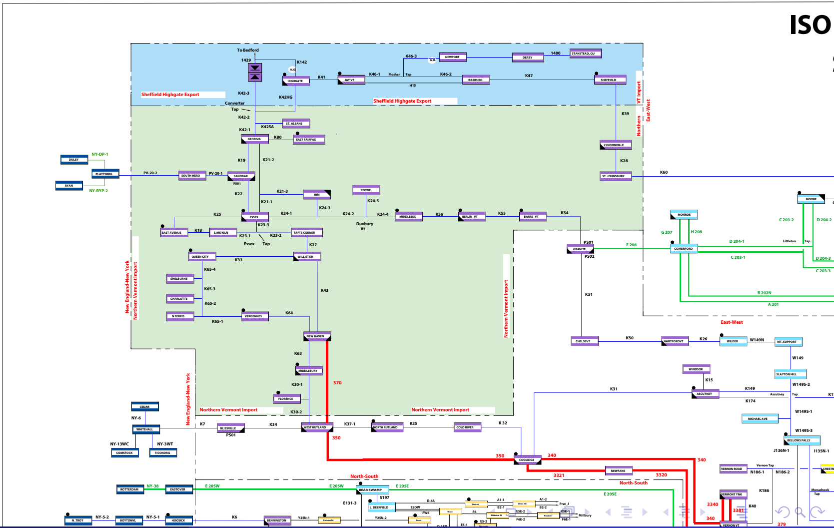

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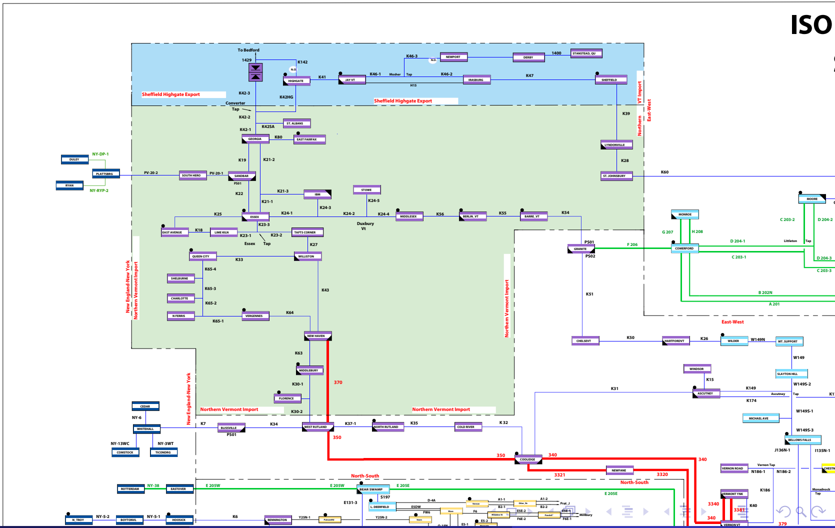

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- Metric: Line failure probability

Notation and Basics

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Variables for Static and Dynamic Analysis

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- Represent voltage $v^{(t)}$ and current $i^{(t)}$ by sinusoidals
- Convert time-domain to phasor: $x^{(t)} = A\cos(\omega t + \delta) = \Re\{Ae^{\mathrm{i}\,\phi}e^{\mathrm{i}\,\omega t}\} = \Re\{\dot{X}e^{\mathrm{i}\,\omega t}\}$

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- Algebraic phasor manipulations (i.e., Ohm, Kirchoff, etc.) depend on a common ω and a reference δ
- Standard to use (ω- and δ-relative) voltage variables: $\tilde{v} := Ve^{\mathrm{i}\,\theta}$ (1)
- Compute scalar (rms) power values for the real and complex components of v and i

Transmission System

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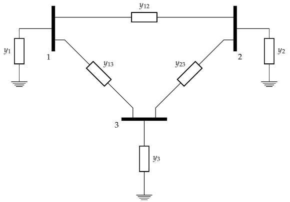
$$S := P + \mathrm{i}\, Q = \tilde{v} \odot [\tilde{Y}\tilde{v}]^*, \quad \text{partitioned as } F(V,\theta) \equiv P, \quad G(V,\theta) \equiv Q \tag{2}$$



$$Y = \begin{pmatrix} y_1 + y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_2 + y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_3 + y_{13} + y_{23} \end{pmatrix}$$

(Wikipedia)

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DAE Model ($\mathcal{M}_{\text{generic}}$)

$$\dot{x} = d(x;\, y), \quad \text{(slower timescale)} \tag{4a}$$

$$0 = pfe(x;\, y), \quad \text{(faster timescale)} \tag{4b}$$

where d represents the generator dynamics and pfe represents the power flow equations

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DAE Model ($\mathcal{M}_{\text{classical}}$), more detail under lossless assumption

$$\dot{\theta}_i = \omega_i - \omega_\mathcal{S}, \qquad i \in \mathcal{G} \cup \mathcal{S} \tag{5a}$$

$$M_{ii}\omega_i = P^i_{\text{net}} - \sum_{j\in\mathcal{B}} V_i V_j B_{ij} \sin(\theta_i - \theta_j) - D_i(\omega_i - \omega_\mathcal{S}), \quad i \in \mathcal{G} \cup \mathcal{S} \tag{5b}$$

$$0 = -P^i_{\text{net}} - \sum_{j\in\mathcal{B}} V_i V_j B_{ij} \sin(\theta_i - \theta_j), \qquad i \in \mathcal{L} \tag{5c}$$

$$0 = Q^i_{\text{net}} - \sum_{j\in\mathcal{B}} V_i V_j B_{ij} \cos(\theta_i - \theta_j), \qquad i \in \mathcal{L} \tag{5d}$$

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Failure Mechanism

For line $\ell = (i, j)$, complex current flow, line energy, and "safe" domain are:

$$\tilde{i}_\ell := (\tilde{v}_i - \tilde{v}_j)\, y_{ij} \tag{6a}$$

$$\Theta_\ell(x) := |\tilde{i}_\ell|^2 = \tilde{i}_\ell \tilde{i}_\ell^* = b_{ij}^2 \left(V_i^2 - 2 V_i V_j \cos(\theta_i - \theta_j) + V_j^2 \right) \tag{6b}$$

$$D := \{x : \Theta_l(x) < \Theta_l^{\max}\}. \tag{6c}$$

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DAE Model ($\mathcal{M}_{\text{Hamiltonian}}$) [?, ?]

$$\begin{bmatrix} \dot{\omega} \\ \dot{\theta} \\ 0 \end{bmatrix} = (J - S) \begin{bmatrix} \nabla_{\omega} \mathcal{H}(\omega, \theta, V; y) \\ \nabla_{\theta} \mathcal{H}(\omega, \theta, V; y) \\ \nabla_{V} \mathcal{H}(\omega, \theta, V; y) \end{bmatrix} \tag{7a}$$

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for a scalar function $\mathcal{H}^y$ (parametrized by y)

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$$\nabla_{\omega}\mathcal{H}^y := M\omega_{\mathcal{V}_{G\cup S}} \tag{8a}$$

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and where J and S are appropriate skew-symmetric and diagonal "structure" matrices representing network interconnection and damping, respectively

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DAE Model Issues

- Above DAE models are not globally well-posed [?]
- DAEs are harder to simulate than ODEs
- Perhaps we can relax the power flow constraint assumption

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Relax the fast dynamics:

$$\dot{x} = A\nabla\mathcal{H} \equiv \begin{cases} \dot{\omega} & = -A_{\omega\omega}\nabla_{\omega}\mathcal{H} - A_{\omega\theta}\nabla_{\theta}\mathcal{H} \\ \dot{\theta} & = A_{\omega\theta}\nabla_{\omega}\mathcal{H} - A_{\theta\theta}\nabla_{\theta}\mathcal{H} \\ \dot{V} & = A_{VV}\nabla_{V}\mathcal{H} \end{cases} \tag{9a}$$

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$$A := \begin{bmatrix} -M^{-1}_{\mathcal{V}_{G\cup S}} D_{\mathcal{V}_{G\cup S}} M^{-1}_{\mathcal{V}_{G\cup S}} & -M^{-1}_{\mathcal{V}_{G\cup S}} T_1^\top & 0 \\ M^{-1}_{\mathcal{V}_{G\cup S}} T_1^\top & -T_2^\top D_{\mathcal{V}_L} T_2^\top & 0 \\ 0 & 0 & D_V^{-1} I_{\mathcal{V}_L} \end{bmatrix} \tag{10}$$

where T_1, T_2 are structure matrices

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Path integral from equilibrium $x^{(0)} = \bar{x}$ to time T:

$$\mathcal{H}^y(x^{(0)}, x^{(T)}) := \int_{(0,\, \theta^0_{\mathcal{V}_{G\cup L}}, V^0_{\mathcal{V}_L})}^{(\omega^{(T)}_{\mathcal{V}_{G\cup S}}, \theta^{(T)}_{\mathcal{V}_{G\cup L}}, V^{(T)}_{\mathcal{V}_L})} \left\langle \begin{bmatrix} \nabla_\omega \mathcal{H}^y \\ \nabla_\theta \mathcal{H}^y \\ \nabla_V \mathcal{H}^y \end{bmatrix}, \begin{bmatrix} \mathrm{d}w \\ \mathrm{d}a \\ \mathrm{d}v \end{bmatrix} \right\rangle \tag{11}$$

Scalar Potential

Path-independent scalar potential [?, ?, ?, ?]:

$$\begin{aligned}\mathcal{H}^y(x^{(0)}, x^{(T)}) = \frac{1}{2}(\omega^{(T)})^\top M_{\mathcal{V}_{G\cup S}}\, \omega^{(T)} + \frac{1}{2}\tilde{v}^{\mathrm{H}}_{\mathcal{V}} B\, \tilde{v}_{\mathcal{V}} + \langle [P_{\text{net}}]^{(0)}_{\mathcal{V}_{G\cup L}}, \theta^{(T)}_{\mathcal{V}_{G\cup L}} \rangle \\ + \langle [Q_{\text{net}}]^{(0)}_{\mathcal{V}_L}, \log\left(V^{(T)}_{\mathcal{V}_L}\right) \rangle + C\end{aligned} \tag{12}$$

Scalar Potential

- Structure of A: full-rank, negative-semidefinite $(\nabla\mathcal{H}(\bar{x})^\top A\,\nabla\mathcal{H}(\bar{x}) \le 0$ along any solution trajectory $\bar{x})$
- Equilibrium points can be characterized by $\nabla\mathcal{H}(x) = 0$
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$$\underset{x}{\text{minimize}} \quad \mathcal{H}(x) \tag{14a}$$

$$\text{s.t.} \quad \Theta_\ell(x) = \Theta_\ell^{\text{max}} \tag{14b}$$

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Definition (Minimizers)

$$\bar{x} := \arg\min \textbf{??} \tag{15a}$$

$$x^\star := \arg\min \textbf{??} \tag{15b}$$

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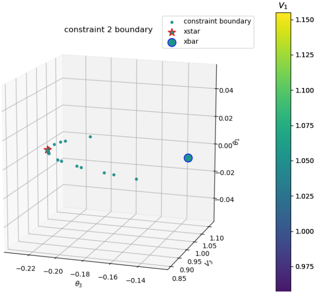

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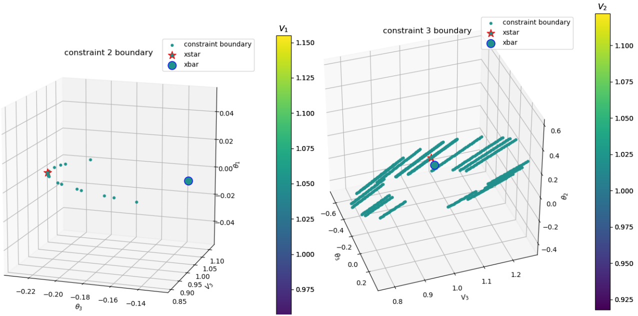

Local energy surface, *left*: discretized constraint surface defined by line-2, *right*: line-3

Energy Landscape

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Phase-space energy surface: ldt

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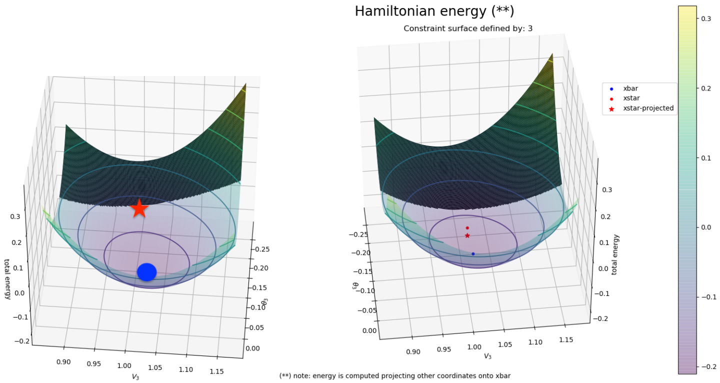

Local energy surface, *left*: energy and constraint surface (black) defined by line-2, *right*: line-3

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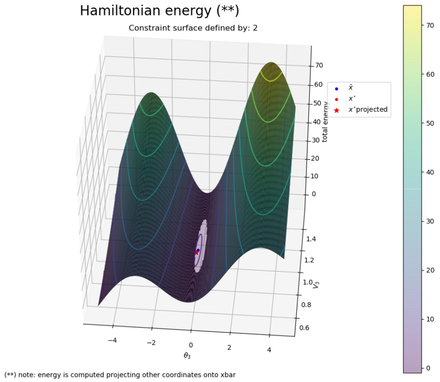
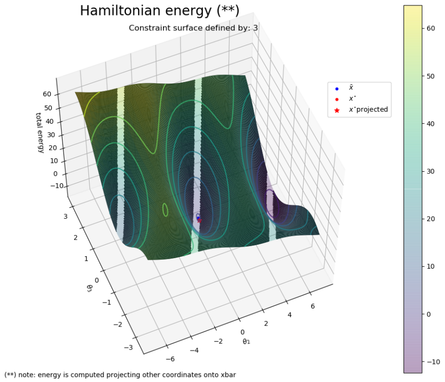
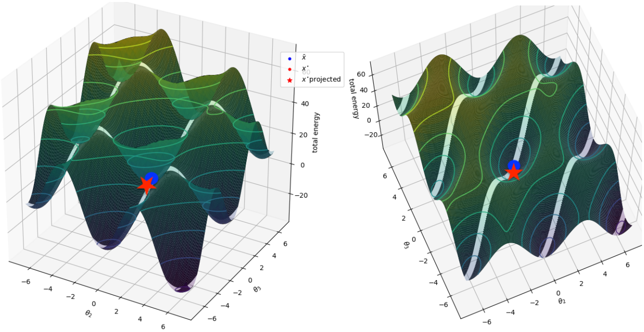




Wider energy surface, *left*: energy and constraint surface (black) defined by line-2, *right*: line-3

Energy Landscape

Phase-space energy surface: ldt



Global energy surface, *left*: energy and constraint surface (black) defined by line-3, *right*: line-3 different view

SDE Formulation

Stochastic Perturbations [?]

- Represent load fluctuations as Gaussian perturbations

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- Guided by saddle-points and potential energy hurdle energy-landscape

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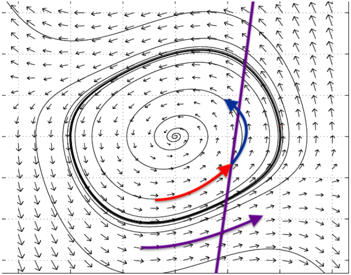
- Reaction rate is governed by the law with:

$$\lim_{\tau \to 0^+} \tau \mathbb{E}\left[T^{\tau}_{\partial D}\right] = \min_{x \in \partial D'} V(\bar{x}, x) \qquad (22)$$

Irreversible Escape Rate Approximation

Crossing Assumptions

1. Non-characteristic constraint boundary (correct "direction" of crossing)
2. $n(x)^\top S\, n(x) > 0$ for $x \in \partial D$ and $n(x)$ the constraint's unit normal (noise in the direction of the constraint)



Crossing events adapted to Van der Pol system portrait[3]

Bruzelius, 2003

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Escape Rate [?, ?]

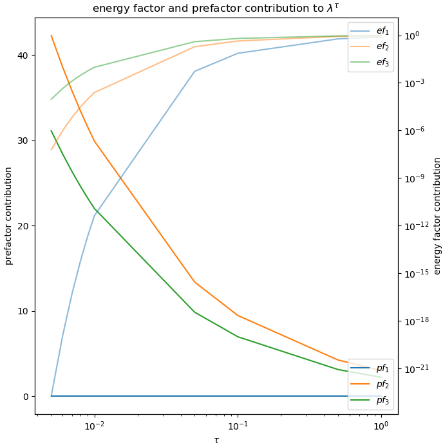
Idea is to use a Laplace approximation around the constrained minimizer:

$$\lambda^\tau = \int_{z \in \partial D} \sqrt{\frac{\det \text{Hess}\, \mathcal{H}(\bar{x})}{(2\pi\tau)^d}}\, e^{\left\{-\frac{V(\bar{x}, z)}{\tau}\right\}} \langle b^y(z), \nabla n(z) \rangle \,\mathrm{d}z \quad (23)$$

$$\underset{\tau \to 0}{\approx} \gamma \left(\nabla_x \mathcal{H}(x^\star)\right)^\top S\, \nabla_x \mathcal{H}(x^\star) \sqrt{\frac{\det \text{Hess}\, \mathcal{H}(\bar{x})}{2\pi\tau B}} \exp\left\{-\frac{\mathcal{H}(x^\star) - \mathcal{H}(\bar{x})}{\tau}\right\} \quad (24)$$

where B captures curvature and volume properties at $x^\star$

Contribution of Prefactor



Computation of λ^τ for various temperatures; split into prefactor (polynomial) and energy factor (exponential)

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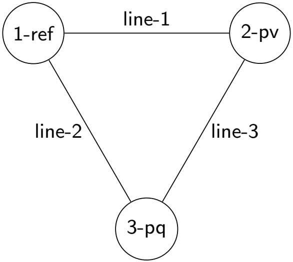
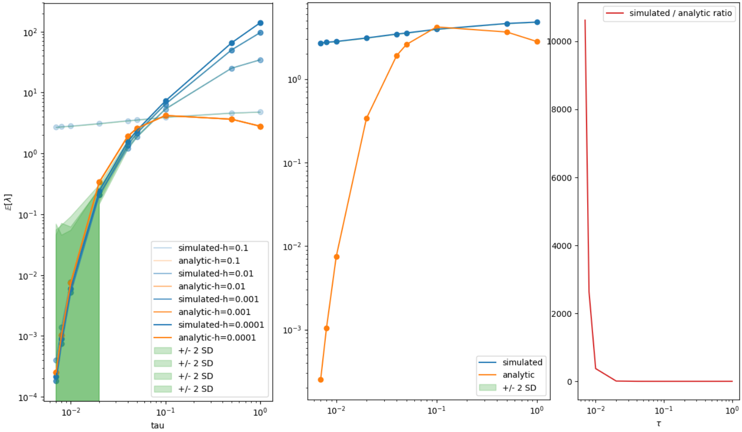


left: 3bus structure from [**?**]

Integration Verification for Failure Rate

Setting integration parameters (step-size):



Inverse of average failure time (1/sec) over 5,000 failures (25 experiments of 200 failures) for 3bus model line 2 at different step sizes; as step-size decreases, we observe low-noise asymptotic agreement between simulated and analytic approximations of $\mathbb{E}[\lambda]$

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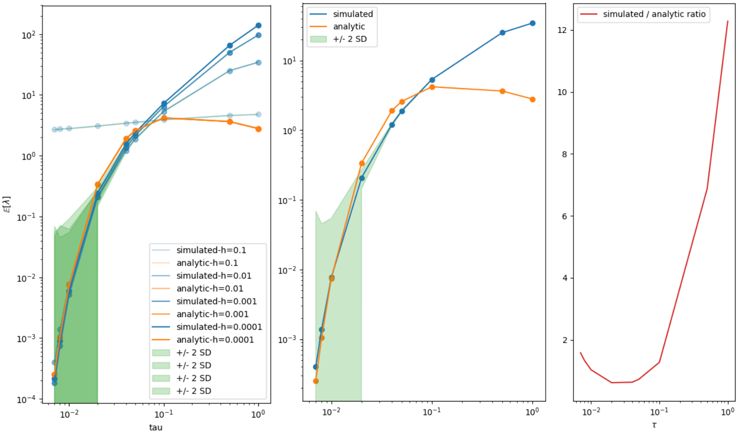

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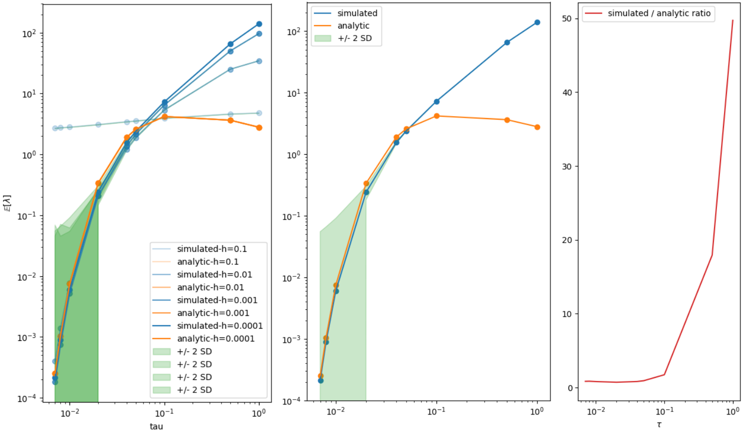

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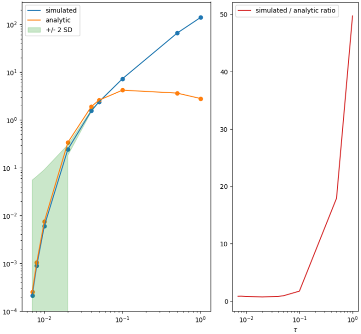
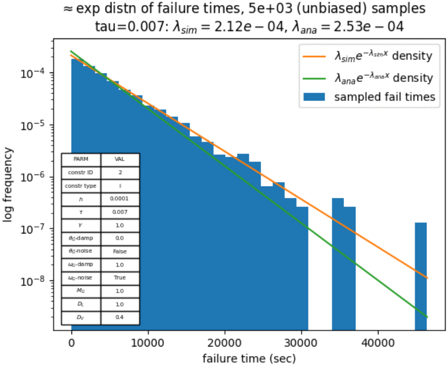
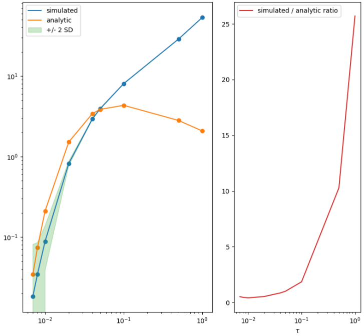
Failures Across Lines (line-2)

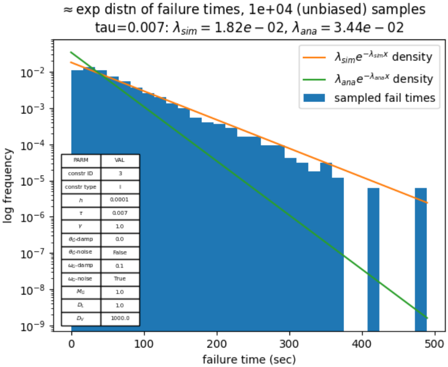




left: inverse of average failure time (1/sec) over 5,000 failures (25 experiments of 200 failures),
right: approximate exponential distribution of failure times

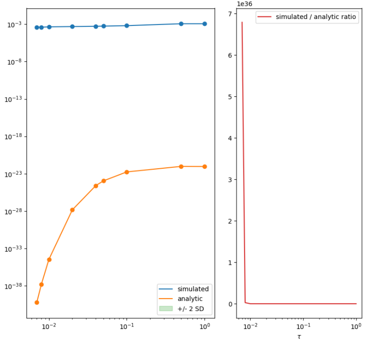
Failures Across Lines (line-3)

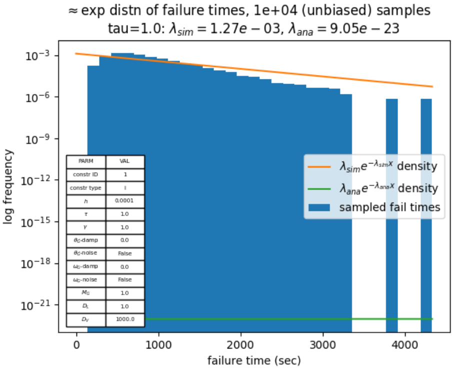




left: inverse of average failure time (1/sec) over 10,000 failures (50 experiments of 200 failures), *right*: approximate exponential distribution of failure times

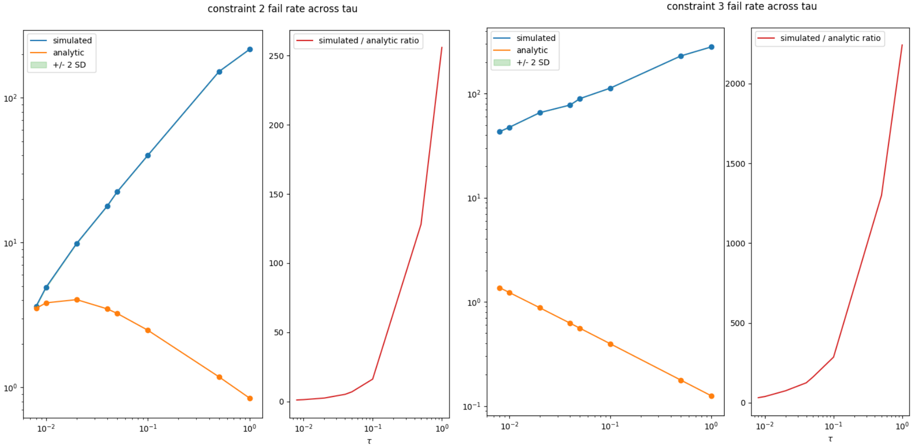
Failures Across Lines (line-1)





note: λ is calculated excluding the prefactor. *left*: inverse of average failure time (1/sec) over 10,000 failures (50 experiments of 200 failures), *right*: approximate exponential distribution of failure times

Failures Across Line Limits



left: line-2 failure rate across line limit Θ_2^{max}, *right*: line-3 failure rate across line limit Θ_3^{max}

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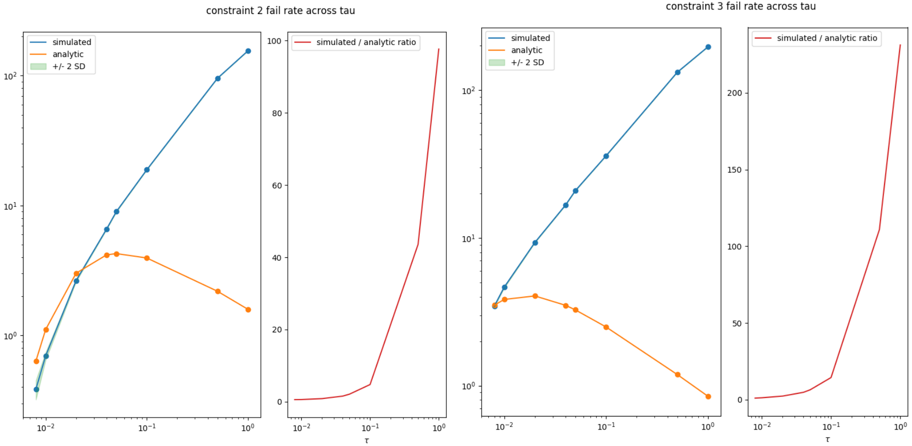

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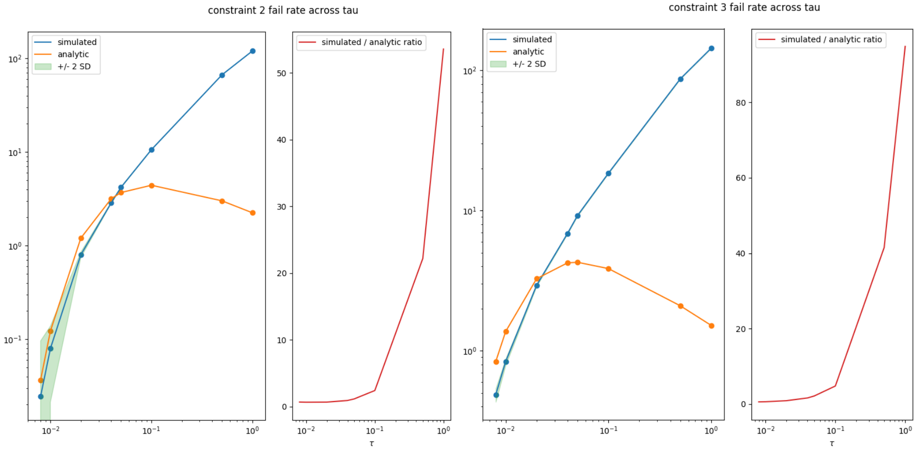

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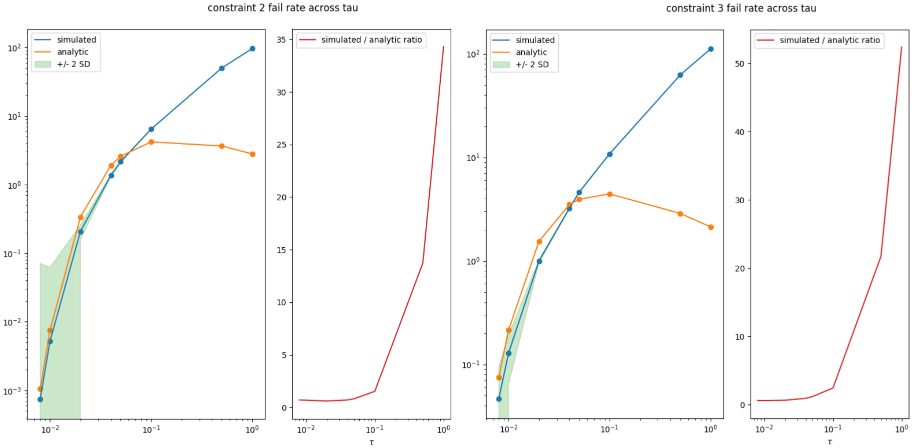

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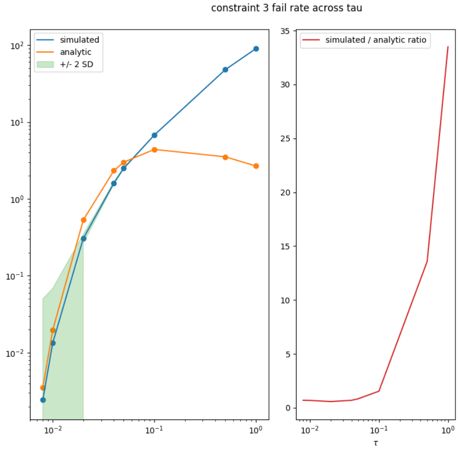

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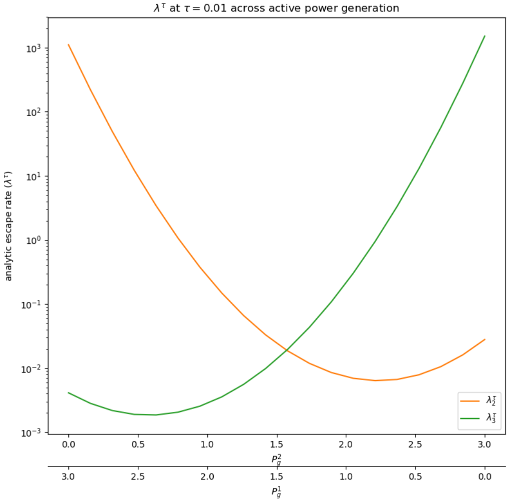
Animations

animations

Sensitivity to Dispatch



Line-2 and line-3 failure rate versus active power generatrion P_g^2

Validation Summary

Takeaways

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- Heuristic measure is when exit points cluster around $x^\star$, not just good agreement between analytic and simulated λ
- Prefactor correction is important; escape point is not a true saddle point

Conclusions

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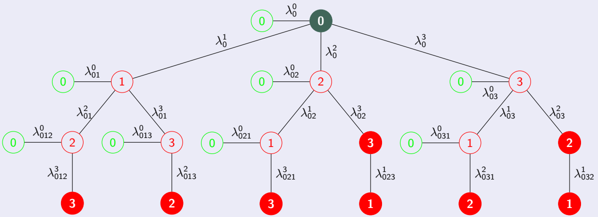
- Failure sequences (parallelize $x^\star$ calculation) and build Markov network



References I