Cascade-constrained ACOPF

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Outline

Introduction

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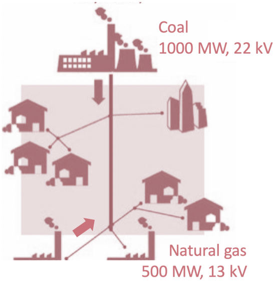
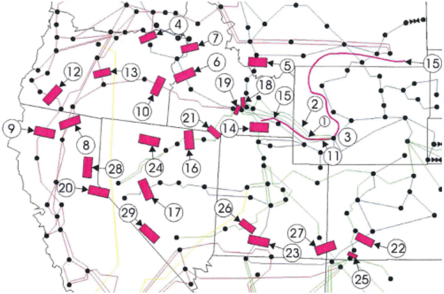
Probability model

Optimization model

Numerical experiments

Motivation





Scheduling electrical generation

- Critical for safe planning and operation of the grid
- Growing complexity of the grid makes this problem more pronounced

Challenges

- Set points can be susceptible to failure in ways that are not well understood
- Component outages don't propagate locally in grid topology
- Need to resolve complex interactions amongst components (dynamics, ACPF)

Existing approaches

Cascade models

Cascade mitigation

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Difficulty: Hard to incorporate in optimization; not real time
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- Protective (before a contingency): ensure grid security against a list of particular contingencies ($N-1$, $N-k$)

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Cascade mitigation

- Protective (before a contingency): ensure grid security against a list of particular contingencies $(N-1, N-k)$
- Corrective (after a contingency): generation re-dispatch, load shed, protective islanding

Existing approaches (cont'd)

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$$\mathcal{R}\colon\ y \to \mathcal{R}(y)$$

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Proposed approach

Thermodynamics-inspired protective model

- Explicit surrogate for cascade risk $\mathcal{R}$ based on the operating state's "potential energy" difference
- Incorporate $\mathcal{R}$ into ACOPF and influence the energy surface by the changing generation schedule

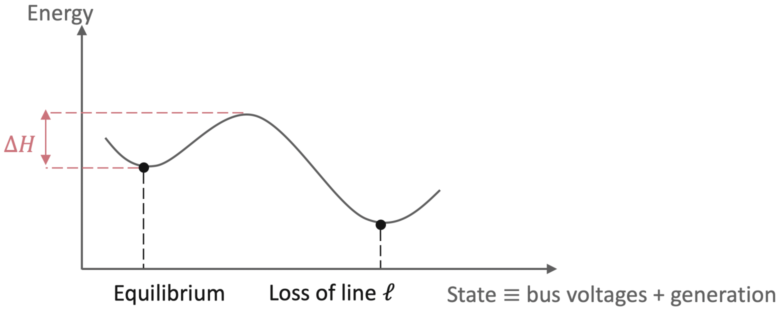

Figure: Energy landscape schematic.

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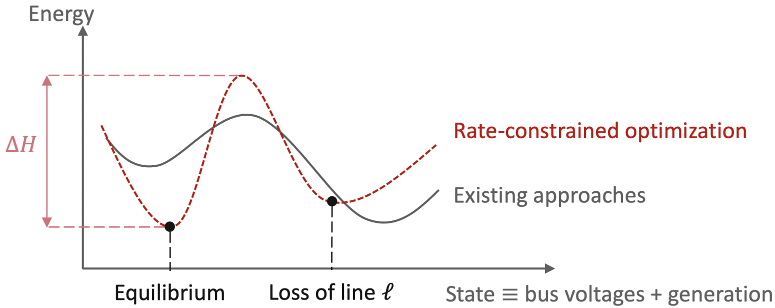

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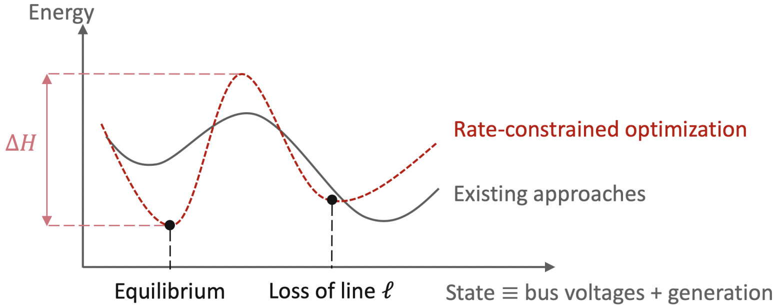

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Goal: limit the probability of single-component failures within a control horizon (similar to $N - 1$)

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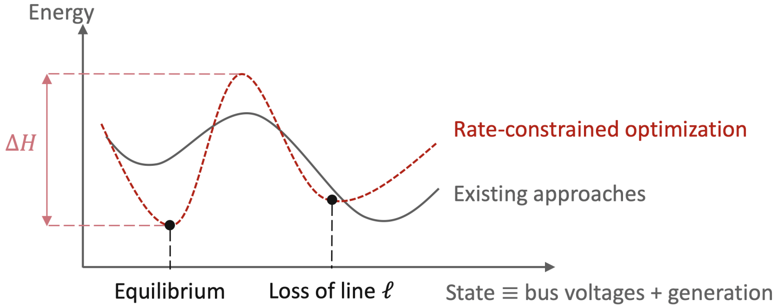

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Question: how effective is controlling the risk of single-component failure for controlling cascade risk?

Proposed approach (cont'd)

Roadmap:

1. "Lower level" state space, model, and optimization (`Probability model` section)
2. "Upper level" state space, model, optimization, and merging of lower level model (`Optimization model` section)
3. Numerical experiments using "KMC" a related cascade tool to simulate cascades based on a similar notion of system "energy" (`Numerical experiments` section)

Probability model

Power transmission network model

Model

- $\mathcal{N} = \{1, 2, \dots n_b\} = \mathcal{N}' \cup \mathcal{G}$ denotes the set of non-generator and generator buses and
- $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ denotes the set of transmission lines
- System state space $\subseteq \mathbb{R}^{2n_b}$ where each bus i has
  - two "unknowns": V_i and θ_i
  - two equations: $p_{net,i} = \hat{p}_{net,i}(V, \theta)$ and $q_{net,i} = \hat{q}_{net,i}(V, \theta)$
  - $p_{net,i} := p_{d,i} - p_{g,i}$ and $q_{net,i} := q_{d,i} - q_{g,i}$

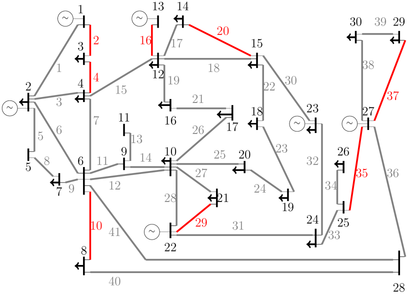

Figure: IEEE 30-bus system

Assumptions

- V_i and $p_{g,i}$ are controllable if i is a generator
- $p_{d,i}$ and $q_{d,i}$ are known if i is a non-generator
- Lossless transmission lines + ACPF equations
- Balance eq. $\sum_i p_{net,i} = 0$ and a "slack" bus at index $i = \sigma$

Power transmission network model

- State space: collect V_i at non-generator buses and θ_i at non-slack buses in the state vector

$$x = \left(\{V_i\}_{i \in \mathcal{N}'}, \{\theta_i\}_{i \in \mathcal{N} \setminus \{\sigma\}}\right) \in \mathbb{R}^d, \quad d = |\mathcal{N}'| + |\mathcal{N}| - 1 \qquad (1)$$

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- Parameters: collect the remaining voltages and net active and reactive powers

$$y = \left(\{V_i\}_{i\in\mathcal{N}\setminus\mathcal{N}'}, \theta_\sigma, p_g, q_g\right) \in \mathbb{R}^m, \quad m = 4n_b - d \qquad (2)$$

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- How x changes over time is described by an SDE

$$\mathrm{d}x_t^\tau = (J - S)\nabla_x \mathcal{H}(x_t^\tau, y)\mathrm{d}t + \sqrt{2\tau S}\mathrm{d}W_t \qquad (3)$$

for an "energy-function" $\mathcal{H} : \mathbb{R}^d \to \mathbb{R}$, system quantities J, S, and Brownian motions $\mathrm{d}W_t$ with variance τ

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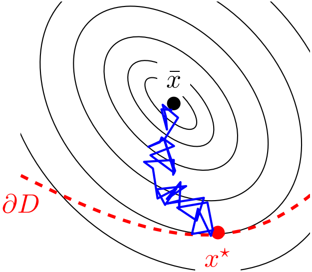
- Energy function (first integral of dynamics)

$$\mathcal{H}(x, y) := \tfrac{1}{2}(V \circ e^{\mathrm{i}\theta})^* \, Y \, (V \circ e^{\mathrm{i}\theta}) + p_{net}^\top \theta + q_{net}^\top \log(V) \qquad (4)$$

Failure model

- Any local energy minimizer $\bar{x}(y) \coloneqq \arg\min_{x \in \mathbb{R}^d} \mathcal{H}(x, y)$ solves the power flow equations and can be taken as a dispatch point
- Query the system for the first time that an observable $(\Theta(x_t^\tau, y))$ of the system exceeds a threshold $(\Theta^{\max})$
- Phrase as a mean first-passage time problem

$$T_{\partial D}^{\tau} \coloneqq \inf\{t > 0 : \Theta(x_t^\tau, y) \geq \Theta^{\max}\}$$



Failure model (cont'd)

We can consider $\Theta(x, y)$ to denote various quantities of interest, for example...

- ...under-voltage violation: $\Theta(x, y) = -V_i$, $\Theta_i^{\max} = -V_i^{\text{trip}}$
- ...exceedance of apparent power flow rating: $\Theta_l(x, y) = (s_{p,l})^2 + (s_{q,l})^2, \;\; \Theta_l^{\max} = (s_l^{\text{trip}})^2$
- ...exceedance of current flow (I) rating for line $l = (i, j)$: $\Theta(x, y) := |B_{i,j}|^2 \left(V_i^2 + V_j^2 - 2V_iV_j \cos(\theta_i - \theta_j)\right), \;\; \Theta_l^{\max} = (I_l^{\text{trip}})^2$

And each describe a safety region $D(y)$, failure boundary $\partial D_l(y)$, and failure region $D_l^{\complement}(y) := \mathbb{R}^d \backslash D_l(y)$ for

$$D_l(y) := \{x \in \mathbb{R}^d : \Theta_l(x, y) < \Theta_l^{\max}\}$$
$$\partial D_l(y) := \{x \in \mathbb{R}^d : \Theta_l(x, y) = \Theta_l^{\max}\}.$$

Computing $\mathcal{R}(y)$ in closed form

- Specifies distribution of failure times $T^{\tau}_{\partial D} \sim \mathrm{Exp}(\lambda)$ governed by failure rate parameter $\lambda = 1/\mathbb{E}[T^{\tau}_{\partial D}]$

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$$\lim_{\tau \to 0} \tau \log \mathbb{E} T^{\tau}_{\partial D_l} = \min_{x \in \partial D_l} \mathcal{H}(\bar{x}(y), y) - \mathcal{H}(x^{\star}(y), y), \qquad (5)$$

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- Can compute subexponential correction to λ, by a Laplace approximation at the "most-likely" failure point

$$x^{\star}_l(y) \coloneqq \arg\min_{x \in \mathbb{R}^d} \left\{ \mathcal{H}(x, y) : \Theta_l(x, y) = \Theta^{\max}_l \right\}, \tag{6}$$

Failure rate for line l

Failure rate: ◂ computation ◂ determinants

$$\lambda_l^\tau(y) \underset{\tau\to 0}{\sim} \mathsf{pf}_l(y) \times \mathsf{ef}_l(y) \tag{7}$$

$$\mathsf{pf}_l(y) := \nabla^\top \mathcal{H}_l^\star(y) S \nabla \mathcal{H}_l^\star(y) \sqrt{\frac{\det \nabla_{xx}^2 \mathcal{H}(\bar{x}, y)}{2\pi\tau B_l^\star(y)}} \tag{8}$$

$$\mathsf{ef}_l(y) := \exp\left[-\frac{\mathcal{H}(x_l^\star, y) - \mathcal{H}(\bar{x}, y)}{\tau}\right], \tag{9}$$

where $\nabla\mathcal{H}_l^\star(y)$ and $B_l^\star(y)$ (a factor accounting for the curvature of $\partial D_l(y)$ in the vicinity of $x^\star$) are given by:

$$\nabla\mathcal{H}_l^\star(y) := \nabla_x \mathcal{H}(x_l^\star, y) \tag{10}$$

$$B_l^\star(y) := \nabla^\top \mathcal{H}_l^\star(y)\, \mathrm{adj}\,(L_l(y))\, \nabla\mathcal{H}^\star(y) \tag{11}$$

$$L_l(y) := \nabla_{xx}^2 \mathcal{H}(x_l^\star, y) - \mu_l \nabla_{xx}^2 \Theta_l(x_l^\star, y), \tag{12}$$

and $\mu_l \in \mathbb{R}$ is the Lagrange multiplier in the constraint defining $x^\star$

Risk for line l

Risk:

$$\mathcal{R}(y) := \mathbb{P}(T^{\tau}_{\partial D_l}(y) \le t_H) = 1 - \exp\left[-\lambda^{\tau}_l(y) t_H\right] \qquad (13)$$

for time horizon of interest t_H

Optimization model

Conceptual outline

- Each y (implicitly) determines the values of x through the power flow equations
- Introduce "lower level" optimization variables x to make this relationship explicit
- Determine the best vector y subject to constraining the probability of line failure, shedding load if required

Optimization formulation

We begin with traditional ACOPF

$$\min_{x,y} \quad c(y) = \sum_{k \in \mathcal{G}} c_k(p_{g,k}) \tag{14a}$$

$$\text{s.t.} \quad V_i^{\min} \leq V_i \leq V_i^{\max} \;\; \forall i \in \mathcal{G} \tag{14b}$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \;\; \forall i \in \mathcal{N} \backslash \mathcal{G} \tag{14c}$$

$$\theta_i^{\min} \leq \theta_i \leq \theta_i^{\max} \;\; \forall i \in \mathcal{N} \tag{14d}$$

$$\Theta(x, y) \leq (I^{\max})^2 \;\; \forall l \in \mathcal{L} \tag{14e}$$

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& \Theta(x, y) \le (I^{\max})^2 \;\; \forall l \in \mathcal{L} && \text{(14e)}
\end{aligned}
$$

Next specify a time horizon t_H and risk threshold ϵ^{lim} to constrain the risk for each line $l \in \mathcal{L}$

$$
\mathcal{R}(y, x(y)) = \mathbb{P}(T^{\tau}_{\partial D_l}(y) \le t_H) = 1 - \exp\left[-\lambda_l^{\tau}(y) t_H\right] \le \epsilon^{\mathrm{lim}} \qquad (15)
$$

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In principle, we can also explicitly limit the probability of *sequences* of line failures by considering increasingly nested problems...but leads to same combinatorial issues as in $N - k$

Computation

Challenges

1. Solving a nested optimization problem for $x^\star(y)$
2. Encoding constraints with determinants of large matrices (recall the failure rate prefactor prefactor)

Efficient computation steps

1. Reformulate the $\mathcal{R}$ constraint(s)
2. Incorporate the nested $x^\star(y)$ problem into the top level
3. Address high-dimensional determinants
4. Implement practical considerations

Constraint reformulation (step 1)

1. Low rank factorization of Θ_l for fixed line l:

$$\nabla^2_{x,x}\Theta_l(x,y) = Q_l(x,y)C_l(x,y)Q_l^\top(x,y)$$

   where $Q_l(x,y) \in \mathbb{R}^{d\times r}$ (and $r \ll m$) and $C_l(x,y) \in \mathbb{R}^{r\times r}$ diagonal

2. Taylor approximation:

$$\begin{aligned}\mathcal{H}(x_l^\star, y) \approx \mathcal{H}(\bar{x}, y) + \nabla_x^\top \mathcal{H}(\bar{x}, y)(x_l^\star - \bar{x}) \\ + \frac{1}{2}(x_l^\star - \bar{x})^\top \nabla^2_{xx}\mathcal{H}(\bar{x}, y)(x_l^\star - \bar{x})\end{aligned}$$

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Specifically...

$$\nabla^2_{xx}\mathcal{H}(\bar{x}, y)(x_l^\star - \bar{x}) = \mu_l \nabla_x \Theta_l(x_l^\star, y) \tag{16}$$

$$\Theta_l(x_l^\star, y) = \Theta_l^{\max} \tag{17}$$

$$\mu_l \, \rho\,(A_l) < 1, \tag{18}$$

where $A_l := D_l(x_l^\star, y)\, Q_l^\top(x_l^\star, y)\, [\nabla^2_{xx}\mathcal{H}(\bar{x}, y)]^{-1}\, Q_l(x_l^\star, y)$ is a $r_l \times r_l$ matrix.

High-dimensional determinants (step 3)

Compute prefactor determinants prefactor using

1. the Taylor approximation of the energy $\mathcal{H}$
2. the low-rank factorization of the constraint Θ

This can be done by introducing auxiliary optimization variables via a matrix relationship (see Appendix)

Takeaway: able to reduce failure rate sub-expressions to (complicated) closed-form expressions involving 3×3 matrices

Practical considerations (step 4)

To address warm-starts and infeasibility, the solution procedure is split into two phases:

1. load-shed determination
2. add $\mathcal{R}$ constraints and candidate solve

Numerical experiments

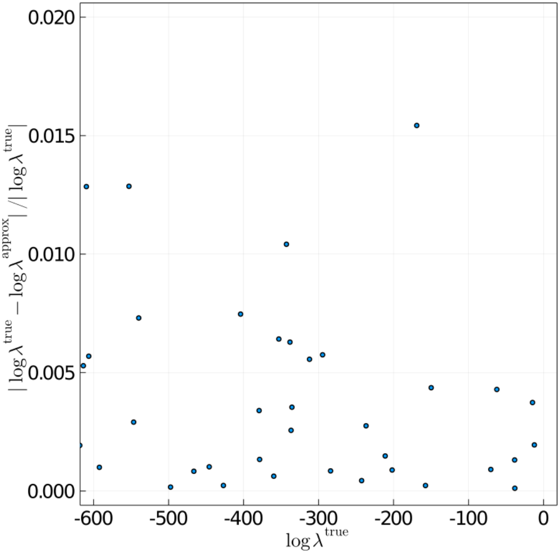
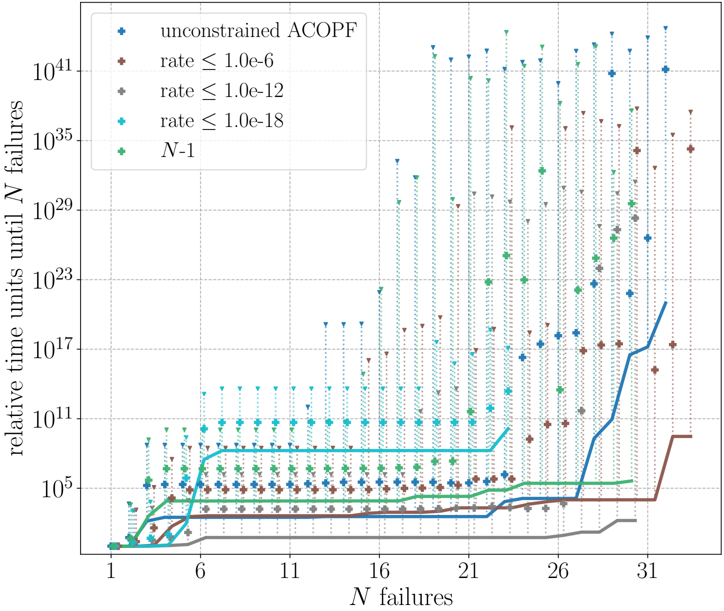


Figure: Validation: KKT-approximated failure rates versus true failure rates for IEEE 118bus model with `PGLib` line limits at traditional ACOPF set-point

Cascade prevention experiment procedure

- Determine a set-point y by enforcing $\mathcal{R} \leq \epsilon^{\lim}$ for various $\epsilon^{\lim}$
- Simulate sequences of line failures from the resulting set point using KMC
- Record time and load-still-served at each failure
- Repeat simulations across a range of parameters settings ($\tau, \Theta^{\max}$, and p_d, q_d load levels); below we show results over load level, known to be an important determinant of cascade risk

$$\tau = 2 \times 10^{-4},\ \Theta^{\max} = 1.04 \times I^{\max},\ p_d = 1.0 \times p_d^0,\ q_d = 1.0 \times q_d^0$$



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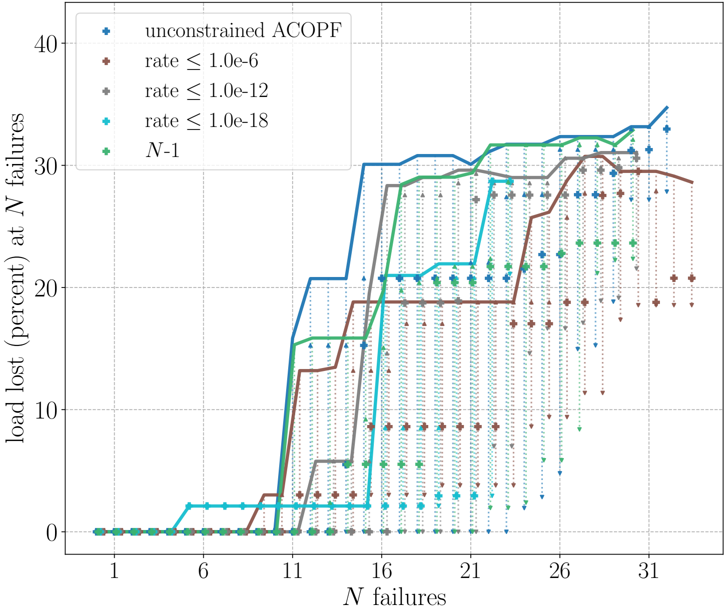
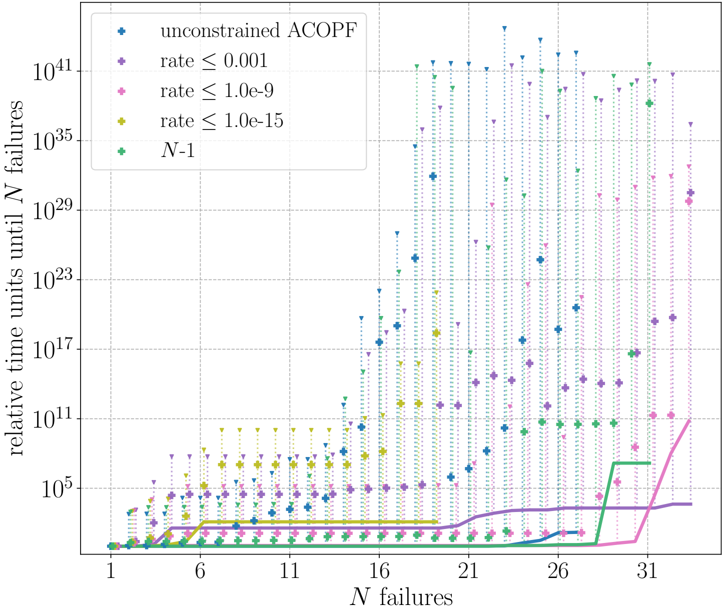
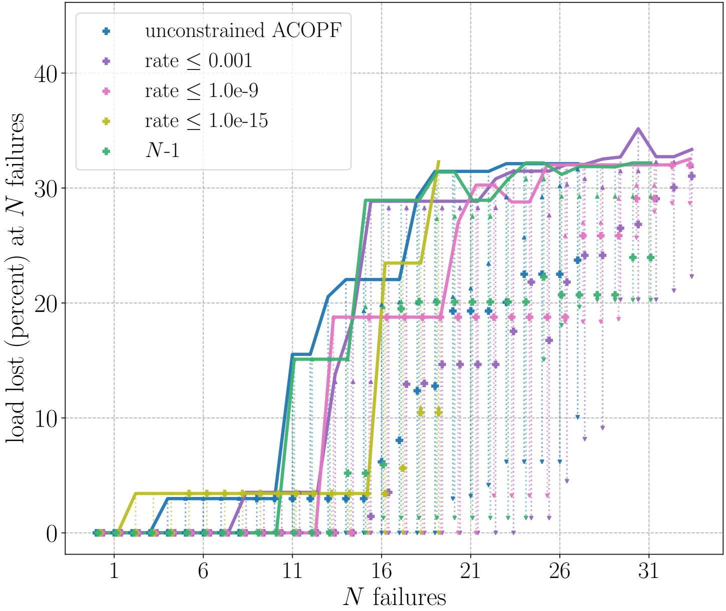


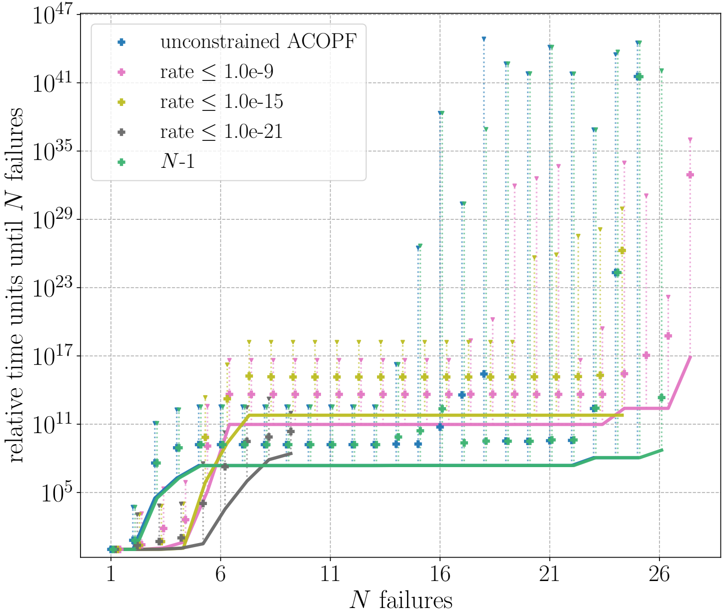
$$\tau = 2 \times 10^{-4},\ \Theta^{\max} = 1.04 \times I^{\max},\ p_d = 1.1 \times p_d^0,\ q_d = 1.1 \times q_d^0$$



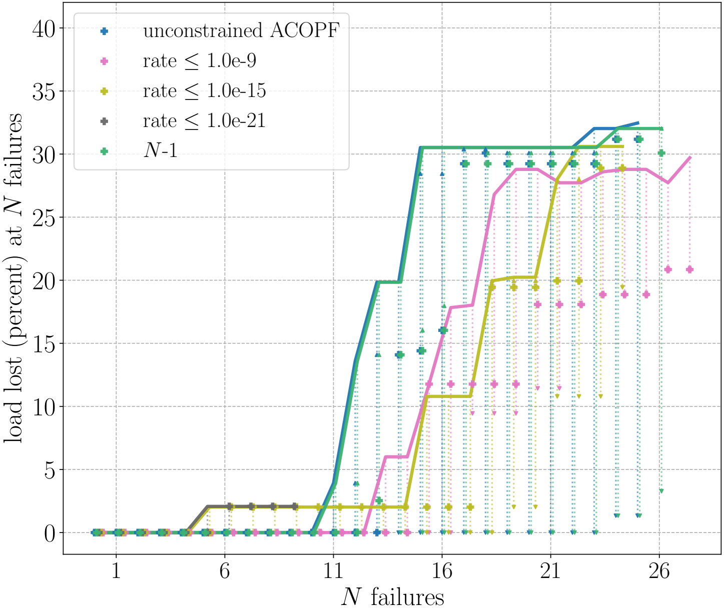
$\tau = 2 \times 10^{-4}, \ \Theta^{\max} = 1.04 \times I^{\max}, \ p_d = 1.1 \times p_d^0, \ q_d = 1.1 \times q_d^0$



$\tau = 2 \times 10^{-4},\ \Theta^{\max} = 1.04 \times I^{\max},\ p_d = 0.9 \times p_d^0,\ q_d = 0.9 \times q_d^0$



$$\tau = 2 \times 10^{-4},\ \Theta^{\max} = 1.04 \times I^{\max},\ p_d = 0.9 \times p_d^0,\ q_d = 0.9 \times q_d^0$$



Shortcomings

- Cascades and failure rates are highly sensitive to τ; cascades are induced by using an artificially high temperature
- `PGLib` line limits are very high, leading to very low initial failure rates; we are in the process of considering an RTS96 case with limits that may reflect reality better

Appendix

- Intermediate terms involved in $\mathcal{R}$ are computable in closed form except for A_l which appears in the definition of $W_l := I - \mu_l A_l$
- Skirt this issue by introducing $Z_l \in \mathbb{R}^{d \times r_l}$ as follows:

$$\nabla^2_{xx} \mathcal{H}(x, y) \cdot Z_l = Q_l(x_l^\star, y), \tag{19}$$

  and then setting $A_l = D_l(x_l^\star, y)\, Q_l^\top(x_l^\star, y)\, Z_l$
- An alternative is to introduce explicit decision variables Z_l in the ACOPF formulation, along with its definition (19) as additional constraints. We implemented this alternative since it allows the complete rate constraint, including all of its intermediate expressions, to be computed analytically.